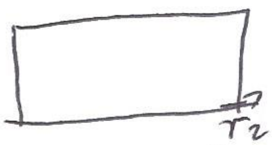
lecture 11 Energy lecture 2

Work: $W = \vec{F} \cdot \vec{r} = F \cdot r \cdot \cos\theta$

$d\vec{r} = \langle dx, dy \rangle$



In general: $W = \int_{\vec{r}_1}^{\vec{r}_2} \vec{F} \cdot d\vec{r} = \int (F_x \cdot dx + F_y \, dy)$

along a path

If there is an acceleration:

$$W = \Delta K = \frac{1}{2} m (v_f^2 - v_i^2)$$

Conservative forces: $W = -\Delta U$

U is the potential energy function of a force.

$\vec{F}_g = -mg\vec{j}$; $\quad U_g = mgy \quad \Delta U = mg(y_2 - y_1)$

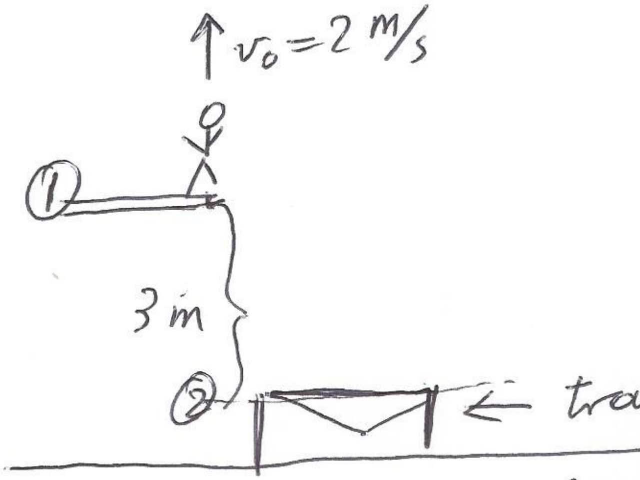
Spring: $\vec{F} = -kx\vec{i} \quad U_s = \frac{1}{2}kx^2$; $\Delta U = \frac{1}{2}k(x_2^2 - x_1^2)$

$\vec{F}$ conservative: $\vec{F} = -\overrightarrow{\text{grad}}\, U$

$$= -\left\langle \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right\rangle \cdot U$$

$$= -\left\langle \frac{\partial U}{\partial x}, \frac{\partial U}{\partial y}, \frac{\partial U}{\partial z} \right\rangle$$

If $\vec{\nabla} \times \vec{F} = \vec{0}$ then $\vec{F}$ is conservative.



- gravity
- spring force

$m = 50\,kg$

← trampoline which acts like a spring with spring constant $k = 0.5\,\frac{N}{m}$

1) $U_1 = mgy =$
$= 50\,kg \cdot 9.8 \cdot 3\, J = 1.47 \cdot 10^3\, J.$

$K_1 = \frac{1}{2} m v_1^2 = 25\,kg \cdot 4 \frac{m^2}{s^2} = 100\, J.$

$U_1 + K_1 = U_2 + K_2$

2) $U_2 = 0$
$K_2 = \frac{1}{2} m v_2^2$

What is mechanical energy conservation?

$$W = \Delta K = \int_{path} \vec{F} \cdot d\vec{r}$$

$$\vec{F} = \vec{F}_{cons} + \vec{F}_{nc}$$

$$W = \Delta K = \underbrace{\int \vec{F}_{cons} \cdot d\vec{r}}_{-\Delta U} + \int \vec{F}_{nc} \cdot d\vec{r}$$

How ~~doe~~ does $\vec{F} = -\vec{\text{grad}}\, U$ come in?

$$W = \Delta K = -\underbrace{\int \vec{\nabla} \cdot U \cdot d\vec{r}} + \int \vec{F}_{nc} \cdot d\vec{r}$$

$$= -\int \left\langle \frac{\partial U}{\partial x}, \frac{\partial U}{\partial y}, \frac{\partial U}{\partial z} \right\rangle \cdot \langle dx, dy, dz \rangle$$

$$= -\int \frac{\partial U}{\partial x} dx + \frac{\partial U}{\partial y} dy + \frac{\partial U}{\partial z} \cdot dz$$

$$= -\int_1^2 dU = -\Delta U = -(U_2 - U_1)$$

$$W = \boxed{-\Delta U + W_{nc} = \Delta K}$$

$$W_{nc} = \Delta K + \Delta U$$

If there is no non-conservative force

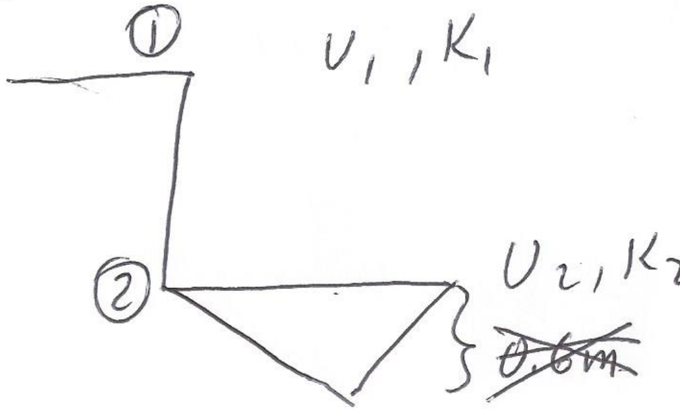
$$W_{nc} = 0$$

$$\boxed{\Delta K + \Delta U = 0}$$

$$K_2 - K_1 + U_2 - U_1 = 0$$

$$\boxed{K_1 + U_1 = K_2 + U_2}$$ law of mechanical energy conservation.

From p2:



$$U_1 + K_1 = U_2 + K_2$$

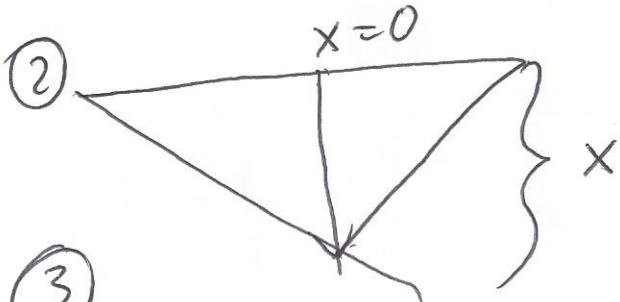
$$\cancel{m} g y_1 + \frac{1}{2}\cancel{m} v_1^2 = \cancel{m} g \overset{0}{\cancel{y_2}} + \frac{1}{2}\cancel{m} v_2^2$$

$$\frac{1}{2} v_2^2 = g y_1 + \frac{1}{2} v_1^2$$

$$v_2^2 = 2 g y_1 + v_1^2$$

$$= 2 \cdot 9.8 \cdot 3 + 4 = 62.8 \ \frac{m^2}{s^2}$$

$$\underline{\underline{v_2 = 7.92 \ \frac{m}{s}}}$$



$$K_2 + U_2 = \frac{1}{2} m v_2^2 + \overset{0}{\cancel{mgy}}$$

$$K_3 + U_3$$

$$= \underbrace{0}_{K_3} + \frac{1}{2} k x^2 + \cancel{0} \underbrace{- mgx}_{U_g}$$

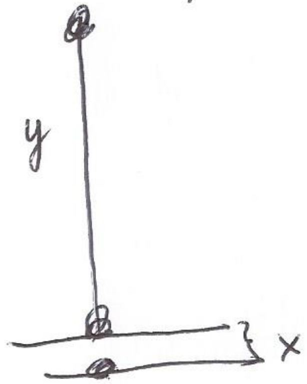
$$\frac{1}{2} m v_2^2 = \frac{1}{2} k x^2 - mgx$$

$$\frac{1}{2} k x^2 - mgx - \frac{1}{2} m v_2^2 = 0$$

$$\Longrightarrow x$$

A rock drops from a 50 m height to the ground and penetrates the ground by 10 cm.

$m = 3$ kg



Non conservative force of friction:

$$W_{nc} = -f_k \cdot x = \Delta U + \underbrace{\Delta K}_{0}$$

$$\Delta U = -mg(y+x)$$

$$= -3 \cdot 9.8 \cdot 50.1 \text{ J}$$

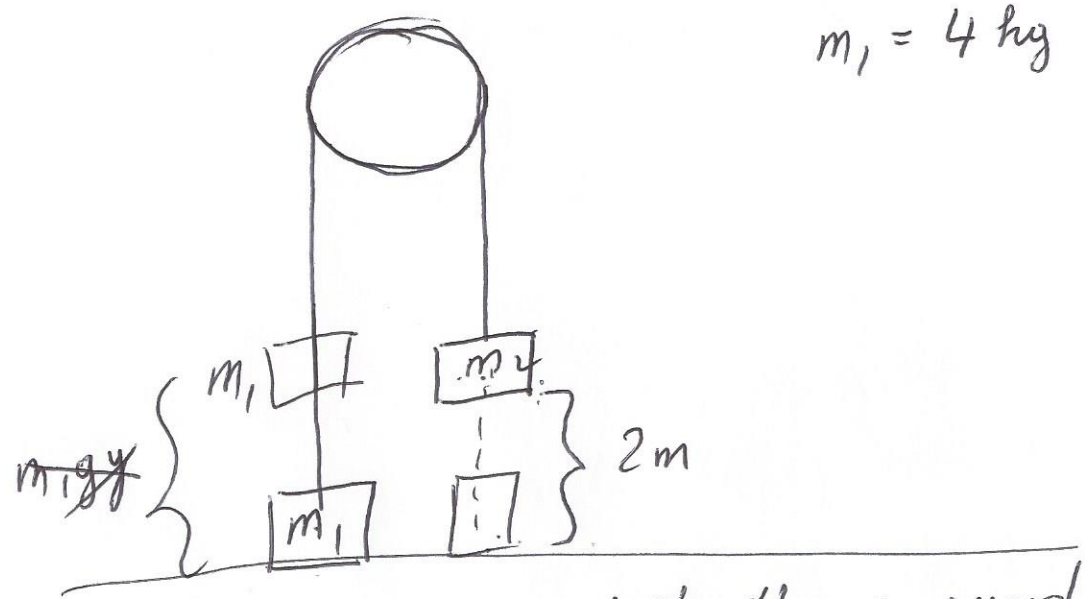
$$=$$

$$+ f_k x = +1.47 \cdot 10^3 \text{ J}.$$

Find f_k

$$f_k = 1.47 \cdot 10^4 \text{ N}$$

$m_1 = 4\,kg \quad m_2 = 5\,kg$



Find v just before m_2 hits the ground

(a) $U_1 + U_2 + K_1 + K_2$

$0 + m_2 g y + 0 + 0$

(B) before m_2 hits the ground

$\cancel{U_2 + U_2}$

$m_1 g y + 0 + \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2$

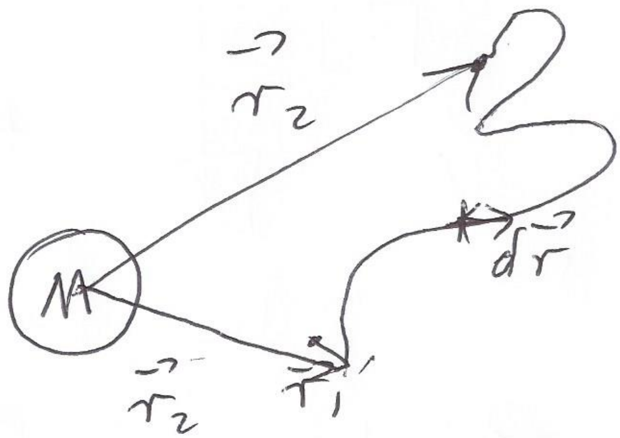
$v_1 = v_2$

$m_2 g y = m_1 g y + \frac{1}{2} m_1 v^2 + \frac{1}{2} m_2 v^2$

$(m_2 - m_1) g y = \frac{1}{2}(m_1 + m_2) v^2$

$$\vec{F} = -\frac{m_1 m_2 G}{r^2}\,\vec{U}_r$$

What is the work done by this force moving from point 1 to point 2 ?



$$W = \int_{\vec{r}_1} -\frac{m_1 m_2 G}{r^2}\,\underbrace{\vec{U}_r \cdot d\vec{r}}_{\text{not parallel necessarily.}}$$

Find curl $\vec{F}$! to determine if $\vec{F}$ is conservative.

<u>another simple example</u> : $\vec{F} = \langle x^2, xy, xyz\rangle$

$$\vec{\nabla} \times \vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \cancel{x^2} & \cancel{xy} & \cancel{xyz} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x^2 & xy & xyz \end{vmatrix} = \vec{C} = \langle C_x, C_y, C_z\rangle$$

$$C_z = \frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y} = \frac{\partial}{\partial x}(xy) - \frac{\partial}{\partial y}(x^2)$$

$$= y - 0$$ not conservative

$$\vec{F} = -\frac{m_1 m_2 G}{(x^2+y^2+z^2)^{3/2}} \langle x, y, z \rangle$$

$$\text{curl } \vec{F} = \vec{C}$$

$$C_x = \frac{\partial}{\partial y} F_z - \frac{\partial}{\partial z} F_y$$

$$\frac{\partial F_z}{\partial y} = \frac{\partial}{\partial y}\left(-\frac{m_1 m_2 G}{(x^2+y^2+z^2)^{3/2}} \cdot z\right)$$